Handout 1

1 Functions

Recall: $y = f(x)$ mean that a function ***f*** uses a variable (an ingredient) ***x*** to make the result ***y***.

Definition: Two sets associated with function, a **domain** (of definition) and a **range**. We denote it $f : A \to B$ and say *f* is a function from (domain) *A* to (range) *B*, if *f* is defined $\forall a \in A$ (for all *a* in *A*) *and* $f(a) \in B, \forall a \in A$.

Another set $C = \{ y \in B : \mathrm{y} = f(x), \mathrm{x} \in \mathrm{A} \} = \{ f(x) : \mathrm{x} \in \mathrm{A} \}$ is called **image** of ***f*.** Note that $C \subset B$.

Note: A function have only one value for each x in the domain.

1.1 Transformation of functions

By applying certain transformations to the graph of a given function, one obtains the graphs of certain related functions, which helps to sketch them quickly by hand.

Transformation			description
Shifting	Shift upward	y = f(x)+c	evaluate f at x and then add c
	Shift downward	y = f(x)-c	evaluate f at x and then subtract c
	Shift right	y = f(x-c)	evaluate f at x-c
	Shift left	y = f(x+c)	evaluate f at x+c
Shrinking & Stretching	Shrink vertically	y = 1/c f(x)	evaluate f at x and then divide by c
	Stretch vertically	y=c f(x)	evaluate f at x and then multiply by c
	Shrink horizontally	y=f(c x)	evaluate f at cx
	Stretch horizontally	y=f(x/c)	evaluate f at x/c
Reflection	Reflect about x-axis	y = -f(x)	evaluate f at x then change the sign
	Reflect about y-axis	y = f(-x)	evaluate f at -x

1.2 Combinations of functions

op name	definition	domain/range	domain/range of result
addition	$(f+g)(x) = f(x) + g(x)$	$f : A \to R, g : B \to R$	$(f+g) : A \cap B \to R$
subtraction	$(f-g)(x) = f(x) - g(x)$	$f : A \to R, g : B \to R$	$(f-g) : A \cap B \to R$
product	$(fg)(x) = f(x)g(x)$	$f : A \to R, g : B \to R$	$(fg) : A \cap B \to R$
division	$(f/g)(x) = f(x)/g(x)$	$f : A \to R, g : B \to R$	$(f/g) : \{x \in A \cap B : g(x) \neq 0\} \to R$
composition	$(f \circ g)(x) = f(g(x))$	$f : A \to R, g : B \to A$	$(f \circ g) : B \to R$

1.3 Inverse functions

Vertical Line Test: A curve in the xy-plane is the graph of function of x if and only if no vertical line intersects the curve more than once.

Definition: A function is a **one-to-one** function if it takes different values for any $x \neq y$, i.e. $f(x) \neq f(y)$.

Horizontal Line Test: A one-to-one function is never intersects with horizontal line more than once.

Definition: A function $f: A \to B$ is an **onto** (B) **function** if its range is its image.

Definition: Let $f: A \to B$ be a function which image is B. A function $g: B \to A$ is said to be an inverse function of f if for every $y = f(x)$ it gets $x = g(y)$. The relationships between *f* and its inverse *g* are reciprocal\mutual; that is, if g the inverse function of f, then f is the inverse function of *g*. One denotes inverse function as $f^{-1}(x)$.

Therefore, we have: $f\left(f^{-1}(x)\right) = x = f^{-1}\left(f(x)\right)$

Corollary: If a function, f is **one-to-one** and **onto** then f^{-1} exists.

Note: $f^{-1}(x) \neq \frac{1}{f(x)} = \left[f(x)\right]^{-1}$

Note: The graph of an inverse function obtained by reflecting the graph off about the line x=y.

Algorithm: How to find inverse function of a given one-to-one function $f(x)$.

1. Write $y = f(x)$
2. Solve this equation above for *x* in term of y (if possible)
3. To express f^{-1} as a function of *x*, interchange *x* and *y*. The resulting equation is $y = f^{-1}(x)$

1.4 Parametric Curves

Definition: A pair of functions $x = f(t)$ and $y = g(t)$ where t considered a parameter, define a parametric curve(as a set of points (x,y)). One can think about regular curve that represent a function as a (degenerated) parametric curve, which has $f(t) = t$.