Course: MA141, Calculus I	Instructor Michael Medvinsky

Handout 3

3 Limits Involving Infinity (∞)

Definition: The line x=a called a vertical asymptote of the curve y=f(x) if at least one of the following statements is true: $\lim_{x\to a^{\pm}} = \infty$, $\lim_{x\to a} = \infty$, $\lim_{x\to a^{\pm}} = -\infty$, $\lim_{x\to a} = -\infty$

Definition: Let f(x) be defined on (a,∞), then $\lim_{x\to\pm\infty} f(x) = L$ means that f(x) gets close to L as x (or –x) gets sufficiently large. We denote the line y=L as horizontal asymptote to y=f(x). In other words for every $\varepsilon > 0$ there is a number $\pm N$ such that $|x| > N$ then implies $|f(x) - L| < \varepsilon$

3.1 The arithmetic of Infinite Limits

1) Given $\lim_{x\to a} f(x) = \infty$ **and** $\lim_{x\to a} g(x) = \infty$**, then**

$\lim_{x\to a}\left(f(x)+g(x)\right) = \infty$

but $\lim_{x\to a}\left(f(x)-g(x)\right) = ???$ need more work

$\lim_{x\to a} f(x)g(x) = \infty$

$\lim_{x\to a} f(x)^{g(x)} = \infty$

but $\lim_{x\to a}\frac{f(x)}{g(x)} = ???$ need more work

$\lim_{x\to a}(\pm p) f(x) = \pm\infty, \quad \mathbb{R} \ni p > 0$

$\lim_{x\to a}\left(f(x)\right)^q = \infty, \qquad \mathbb{Q} \ni q > 0$

2) Given $\lim_{x\to a} f(x) = \infty$ **and** $\lim_{x\to a} h(x) = \pm L, \mathbb{R} \ni L > 0$**, then**

$\lim_{x\to a}\left(f(x)+h(x)\right) = \infty$

$\lim_{x\to a} f(x)h(x) = \pm\infty$

3) Given $\lim_{x\to a} f(x) = \infty$ **and** $\lim_{x\to a}\tilde{h}(x) = 0$ **then**

$\lim_{x\to a} f(x)\tilde{h}(x) = ???$ need more work

$\lim_{x\to a} f(x)^{\tilde{h}(x)} = ???$ need more work

but $\lim_{x\to a}\frac{f(x)}{\tilde{h}(x)} = ???$ need more work