Course: MA141, Calculus I	Instructor Michael Medvinsky

Handout 6

11 Minimum Maximum Values

Definition: Let *f* be a function from domain *D* to range *R*, i.e. $f: D \to R$ and let $c \in \tilde{D} \subseteq D$. If $f(c) \geq f(x)$ for all $x \in \tilde{D}$ then $f(c)$ is the **local maximum** of *f* on *D.* Similarly if $f(c) \leq f(x)$ for all $x \in \tilde{D}$ - $f(c)$ is the **local minimum** of *f* on *D*.

Definition: Let *f* (*x*) be a function from domain *D* to range *R*, i.e. $f: D \to R$ and let $c \in D$. If $f(c) \geq f(x)$ for all $x \in D$ then $f(c)$ is an **absolute/global maximum** of *f* on *D*. Similarly, if $f(c) \leq f(x)$ for all $x \in D$ - $f(c)$ is an **absolute/global minimum** of *f* on *D.*

Extreme Value Theorem: If *f(x)* **continuous** on **closed** interval [a,b], the f(x) attain maximum and minimum on [a,b].

Fermat's Theorem: If *f(x)* has a local maximum or minimum at *c* and if *f'(c)* exists, then *f'(c)*=0.

Definition: Let *f(x)* be defined on D. We say $c \in D$ is a **critical number** of *f(x)* if $f'(c) = 0$ or $f'(c)$ does not exists.

Note: If *f(x)* has a local maximum or minimum at c, then *c* is a critical number of *f(x)*.

Algorithm: To find absolute maximum & minimum of closed interval:

- Find the values of *f* at the critical numbers.
- Find the values of *f* at end points.
- The larger\smaller value obtained in previous steps is the absolute maximum/minimum.

12 Derivatives and the Shapes of Curves

Rolle's Theorem: Let *f* be continuous on $[a,b]$ and differentiable on (a,b). If $f(a) = f(b)$ then there is $c \in (a,b)$ such that $f'(c) = 0$.

La-grange's Mean Value Theorem(MVT) (generalization of Rolle's Thm): Let f be continuous on $[a,b]$ and differentiable on (a,b). There is $c \in (a,b)$ such that $f'(c) = \frac{f(b) - f(a)}{b - a}$.

Definition: Function *f* is **increasing/decreasing** in interval *I* if $x_2 > x_1 \Rightarrow f(x_2) \geq f(x_1)$

($x_2 > x_1 \Rightarrow f(x_2) \leq f(x_1)$). If instead of $\leq, \geq$ one writes <,>, then *f* is **strictly** increasing/decreasing.

Increasing/Decreasing Test: If $f'(x) > 0$ ($f'(x) < 0$) on interval *I* then f(x) is increasing (decreasing) *I*.

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The First Derivative Test: Suppose *c* critical number of continuous function *f*.

- If *f'* changes from positive to negative – *f(c)* is a local maximum
- If *f'* changes from negative to positive– *f(c)* is a local minimum
- If *f'* doesn't changes the sign then *f(c)* has no local max/min.

Concavity:

If *f'* is increasing (decreasing), a.k.a $f''(c)>0$ ($f''(c)<0$) function on an interval *I* then if concave upward (downward) aka smile(sad) on *I*. If f'' changes sign near *c* then there is an inflection point.

The Second Derivative Test:Suppose $f''(x)$ is continuous near *c*

- If $f'(x)=0$ and $f''(c)>0$ then f(c) is local minimum
- If $f'(x)=0$ and $f''(c)<0$ then f(c) is local maximum
- If $f'(x)=0$ and $f''(c)=0$ or doesn't exists – use another method (e.g. FDT)

Algorithm: Function investigation and Sketching Graph of function:

1. Find domain of definition.
2. Find whether the function is even $f(-x)=f(x)$, odd $f(-x)=-f(x)$ or neither.
3. Find the period p of a function $f(x+p)=f(x)$.
4. Find where the function crosses axes, aka solve $f(0)$, $f(x)=0$
5. Find asymptotes, a line $l(x)$ if $\lim_{x\to\pm\infty}\left|f(x)-l(x)\right|=0$ or a line $x=x_0$ if $\lim_{x\to x_0}\left|f(x)\right|=\infty$
6. Do the following derivatives tests:
   a. Find critical points (end points and where $f'(x)=0$ or doesn't exists)
   b. Find where *f* is increasing $f'(x)>0$ or decreasing $f'(x)<0$.
   c. Define the concavity, $f''(x)>0$ smiling upward concavity, $f''(x)<0$ sad downward concavity.
   d. Classify extreme points Using First and/or Second Derivative tests.
7. Sketch the Graph

Cauchy's MVT: Let f, g be continuous on $[a,b]$ and differentiable on (a,b), and assume $g'(x)\neq 0$. There is $c\in(a,b)$ such that $$\frac{f'(x)}{g'(c)}=\frac{f(b)-f(a)}{g(b)-g(a)}$$

The L'Hospital's Rule: The good use of Cauchy MVT is in the L'Hospital's (pronounced Loopeetal) rule, which say if $\lim_{x\to a}\frac{f(x)}{g(x)}$ is of the $\frac{"0"}{"0"},\frac{"\infty"}{"\infty"}$ form then $\lim_{x\to a}\frac{f(x)}{g(x)}=\lim_{x\to a}\frac{f'(x)}{g'(x)}$.