Handout 8

14.1 Properties of Definite Integral

1) $\int_a^b f(x)dx = -\int_b^a f(x)dx$

2) $\int_a^a f(x)dx = 0$

3) $\int_a^b cdx = c(b-a)$

4) $\int_a^b f(x) \pm g(x)dx = \int_a^b f(x)dx \pm \int_a^b g(x)dx$

5) $\int_a^b cf(x)dx = c\int_a^b f(x)dx$

6) $\int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx, \forall c \in \mathbb{R}$

7) $f(x) \geq 0, \forall x \in [a,b] \Rightarrow \int_a^b f(x)dx \geq 0$

8) $f(x) \geq g(x), \forall x \in [a,b] \Rightarrow \int_a^b f(x)dx \geq \int_a^b g(x)dx$

9) $m \leq f(x) \leq M \Rightarrow (b-a)m \leq \int_a^b f(x)dx \leq (b-a)M$

Evaluation theorem: Let *f* be continuous on [a,b] and let *F* be arbitrary antiderivative of *f* , i.e. *F'=f*, then $\int_a^b f(x)dx = F(x)\Big|_a^b = F(b) - F(a)$

14.2 Indefinite Integral

Definition: The common notation of antiderivative is $\int f(x)dx$

$\int f(x)dx = F(x)$ means $F'(x) = f(x)$

Note that the difference between indefinite integral $\int f(x)dx$ and definite integral $\int_a^b f(x)dx$ is that the former is a function whereas the latter is a number. The connection is given by the evaluation theorem: $\int_a^b f(x)dx = \int f(x)dx\Big|_a^b$

Table of indefinite integrals:

$\int cf(x)dx = c\int f(x)dx$	$\int a^x dx = \frac{a^x}{\ln a} + c$
$\int f(x) + g(x)dx = \int f(x)dx + \int g(x)dx$	$\int \sin x = -\cos x + c$
$\int x^n dx = \frac{1}{n+1}x^{n+1} + c$	$\int \cos x = \sin x + c$
$\int \frac{1}{x}dx = \ln x + c$	$\int \tan x dx = -\ln(\cos x) + c$
$\int \ln x dx = x\ln x - x + c$	$\int \cot x dx = \ln(\sin x) + c$
$\int e^x dx = e^x + c$	$\int \frac{1}{1+x^2}dx = \arctan x + c$

Course: MA141, Calculus I	Instructor Michael Medvinsky

$\int \frac{1}{\sqrt{1-x^2}}dx = \arcsin x + c$	$\int \frac{1}{\sqrt{x^2 \pm a^2}}dx = \ln\left(x+\sqrt{x^2 \pm a^2}\right)+c$
$\int \frac{1}{a^2+x^2}dx = \frac{1}{a}\arctan\left(\frac{x}{a}\right)+c$	$\int \frac{A}{x-a}dx = A\ln(x-a)+c$
$\int \frac{1}{a^2-x^2}dx = \frac{1}{2a}\ln\left(\frac{a+x}{a-x}\right)+c$	$\int \frac{A}{(x-a)^n}dx = A\cdot\frac{1}{1-n}(x-a)^{1-n}+c, n>1$
$\int \frac{1}{\sqrt{a^2-x^2}}dx = \arcsin\left(\frac{x}{a}\right)+c$	$\int \frac{Ax+B}{x^2+px+q} = \frac{2B-Ap}{\sqrt{4q-p^2}}\cdot arctan\left(\frac{2x+p}{\sqrt{4q-p^2}}\right)$

The Fundamental Theorem Calculus: Fundamental Theorem of Calculus provides a connection between differentiable and integral calculus.

Suppose f(x) is continuous on [a,b].

1) If $g(x)=\int_a^x f(t)\,dt$ then $g'(x)=f(x)$

2) $\int_a^b f(t)\,dt = F(b)-F(a)$ for any F antiderivative of *f*(*x*), that is $F' = f$

The Substitution Rule, Indefinite Integral: $\int f(g(x))g'(x)dx \underset{\substack{t=g(x)\\ dt=g'(x)dx}}{=} \int f(t)dt$

The Substitution Rule, Definite Integral: $\int f(g(x))g'(x)dx \underset{\substack{t=g(x)\\ dt=g'(x)dx}}{=} \int f(t)dt$

Even and Odd Function:

- If *f* is **even** then $f'(x)$ and $\int f(x)dx$ are **odd** functions and also $\int_{-a}^{a} f(x)dx = 2\int_0^a f(x)dx$
- If *f* is **odd** then $f'(x)$ and $\int f(x)dx$ are **even** functions and also $\int_{-a}^{a} f(x)dx = 0$

Integration be parts: Rewrite Chain Rule $\frac{d}{dx}(fg) = g\frac{df}{dx} + f\frac{dg}{dx}$ as $d(fg) = gdf + fdg$ and then integrate (both sides) to get $\int d(fg) = fg = \int gdf + \int fdg \Rightarrow \int fdg = fg - \int gdf$ or for definite integral $\int_a^b fdg = fg\big|_a^b = \int_a^b gdf$

Special Substitutions:

Substitution	relationship	Useful for
$t=\tan\frac{x}{2}$	$dx = \frac{1}{1+t^2}dt, \cos x = \frac{1-t^2}{1+t^2}, \sin x = \frac{2t}{1+t^2}$	$+,-,\cdot,\div$ of sin and cos
$t=\tan x$	$dt = \left(1+\tan^2 x\right)dx = \frac{dx}{\cos^2 x} = \frac{dx}{1+t^2}$	$+,-,\cdot,\div$ of tan and $\sin^{2n} x, \cos^{2n} x$