PRACTICE Test I Solution

MA141-008 09/12/2018 **Name:** ________________________

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Read all of the following information before starting the exam:

- Complete each problem. Show all of your work clearly and in order and justify your answers, as partial credit will be given when appropriate and there may be NO credit given for problems without supporting work. **Circle or otherwise indicate your final answers your final answers.** All answers should be completely simplified, unless otherwise stated.
- You may copy your final answers to this sheet, but remember that you will be graded for your work and not for the final answer.

  You may not use calculator.

1. Find the domain of $\frac{1}{\sin x - 1}$

   Solution: The function is defined everywhere but at $x \in A$, where
   $$A = \{\sin x = 1\} = \{x = \frac{\pi}{2} + 2\pi n \mid n = \pm 1, \pm 2, \pm 3, \dots\}$$
   Therefore the domain is $\mathbb{R} \setminus A$

2. Find the inverse functions to the following equations and state the domain D and range R of both:

   (a) $f(x) = 10e^{-x^3}$ $\quad f^{-1}(x) =$ ____________, $D =$ ____________, $R =$ ____________

   Solution:
   $$f : D = \mathbb{R}, \quad R = \{y|y > 0\}$$
   $$f^{-1}(x) = \sqrt[3]{\ln(10/x)}$$
   $$f^{-1} : D = \{x|x > 0\}, \quad R = \mathbb{R}$$

   (b) $f(x) = \log_{10}(x^2+10)$ $\quad f^{-1}(x) =$ ____________, $D =$ ____________, $R =$ ____________

   Solution: $f : D = \mathbb{R}, \quad R = \{y|y \geq 1\}$
   Note, that $\log_{10}((3\sqrt{(10)})^2 + 10) = 2 = \log_{10}((-3\sqrt{(10)})^2 + 10)$, therefore the function isn't one-to-one, which means it is non invertible.

   (c) $f(x) = \frac{x}{1+x}$ $\quad f^{-1}(x) =$ ____________, $D =$ ____________, $R =$ ____________

Solution:

$$f: D = \{x \neq -1\}, \quad R = \{y \neq 1\}$$
$$f^{-1}(x) = \frac{x}{1-x}$$
$$f^{-1}: D = \{x \neq 1\}, \quad R = \{y \neq -1\}$$

(d) $f(x) = \frac{1}{1+e^{-x}}$ $\quad f^{-1}(x) =$ ______________, $D =$ ______________, $R =$ ______________

Solution:

$$f: D = \mathbb{R}, \quad R = \{0 < y < 1\},$$
$$f^{-1}(x) = \ln\left(\frac{x}{1-x}\right)$$
$$f^{-1}: D = \{0 < x < 1\}, \quad R = \mathbb{R}$$

3. Find the following limits or show they do not exist. Show your work in detail.

   (a) $\lim_{x\to 1} \frac{x^2+2x-3}{x^2-x} = \lim_{x\to 1} \frac{(x+3)(x-1)}{x(x-1)} = \lim_{x\to 1} \frac{x+3}{x} = 4$

   (b) $\lim_{x\to 0} \frac{\sqrt{1-x^2}-1}{x^2} = \lim_{x\to 0} \frac{1-x^2-1}{x^2(\sqrt{1-x^2}+1)} = \lim_{x\to 0} \frac{-1}{\sqrt{1-x^2}+1} = -\frac{1}{2}$

   (c) $\lim_{x\to -6} \frac{2x+12}{|x+6|} = \lim_{x\to -6^{\pm}} \frac{2(x+6)}{|x+6|} = \pm 2$, i.e. the limit doesn't exists.

4. Use squeeze theorem to find the following limits

   (a) $\lim_{x\to 0} x^2 e^{\sin\frac{1}{x}}$

Solution: Since $-1 \leq \sin y \leq 1$ for any $y \in \mathbb{R}$ this also true for $y = \frac{1}{x} \in \mathbb{R}$. Conclude that $e^{-1} \leq e^{\sin\frac{1}{x}} \leq e$ and therefore

$$0 \leftarrow x^2 e^{-1} \leq x^2 e^{\sin\frac{1}{x}} \leq e x^2 \to 0$$

Thus, by squeeze theorem

$$\lim_{x\to 0} x^2 e^{\sin\frac{1}{x}} = 0$$

   (b) $f(x) = -\frac{x^2}{4} - \frac{x}{2}$ and $h(x) = \frac{x^2}{3} + \frac{2x}{3} + \frac{2}{3}$. What can you say about $\lim_{x\to -1} g(x)$ if it is known that $f(x) \leq g(x) \leq h(x)$ for x near -1.

Solution: We can only bound the limit in this case

$$f(x) \to \frac{1}{4} \leq \lim_{x \to -1} g(x) \leq \frac{1}{3} \leftarrow h(x)$$

Good luck!