PRACTICE Test IV Solution

MA141-008 11/28/2018 **Name:** __________________________
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Read all of the following information before starting the exam:

- Complete each problem. Show all of your work clearly and in order and justify your answers, as partial credit will be given when appropriate and there may be NO credit given for problems without supporting work. **Circle or otherwise indicate your final answers your final answers.** All answers should be completely simplified, unless otherwise stated.

  You may not use calculator.

1. Evaluate the following indefinite integrals

(i) $\int 5x^3 + 6x^2 - 2x + \frac{1}{x} dx =$

Solution:

$$= \frac{5}{4}x^4 + 2x^3 - x^2 + \ln|x| + C$$

(ii) $\int \frac{3x^2+x}{x^3+0.5x^2+5} dx =$

Solution:

$$= \int \frac{du}{u} = \ln u = \ln(x^3 + 0.5x^2 + 5)$$

(iii) $\int x \sin x dx =$

Solution:

$$\int x \sin x dx = \sin(x) - x\cos(x) + C$$

2. Evaluate the following definite integrals

(i) $\int_0^{\pi/2} \cos 3x dx =$

Solution:

$$\int_0^{\pi/2} \cos 3x dx = \frac{\sin 3x}{3}\Big|_0^{\pi/2} = \frac{-1-0}{3} = -\frac{1}{3}$$

(ii) $\int_0^{\left(\frac{\pi}{2}\right)^2} \sin\sqrt{x}dx$

Solution:

$$\begin{aligned}\int_0^{\left(\frac{\pi}{2}\right)^2} \sin\sqrt{x}dx &\underset{2tdt=dx}{\overset{t^2=x}{=}} \int_0^{\pi/2} 2t\sin tdt = -2\int t(\cos t)'\,dt = -2t\cos t + 2\int \cos t = \\ &= (-2t\cos t + 2\sin t)\,|_0^{\pi/2} = 2\end{aligned}$$

(iii) $\int_{\pi/6}^{\pi/3} \frac{\cos 2x-1}{\cos 2x+1}dx =$

Solution:

$$\int_{\pi/6}^{\pi/3} \frac{\cos 2x-1}{\cos 2x+1}dx = \int \frac{-2\sin^2 x}{2\cos^2 x}dx = -\int 1 - \frac{1}{\cos^2 x}dx = -x + \tan x|_{\frac{\pi}{6}}^{\frac{\pi}{3}} = -\frac{\pi}{6} + \frac{3}{2}$$

(iv) $\int_{-1}^{1} \sin\sin xdx =$

Solution:

$$\int_{-1}^{1} \sin(\sin x)dx = 0$$

because $\sin(\sin x)$ is odd function.

3. Approximate the following integral using midpoint rule, divide the interval into 4 subintervals.

$$\int_0^1 \sqrt{x}dx =$$

4. Find the area A between the curves $y = x^2 + 6x - 4$ and $y = -x^2 + x + 8$.

Solution:

The intersection occurs at $x^2+6x-4=-x^2+x+8 \iff -f(x)=2x^2+5x-12=0 \iff x_{1,2}=\frac{-5\pm\sqrt{25+96}}{4}=\frac{-5\pm 11}{4}=-4,\frac{3}{2}$ and we have

$$\begin{aligned} A &= \int_{-4}^{3/2} f dx = -\frac{2}{3}x^3-\frac{5}{2}x^2+12x|_{-4}^{3/2} = \\ &= \overbrace{-\frac{9}{4}-\frac{45}{8}}^{-63/8=-7\frac{7}{8}}+18-\left(\overbrace{\frac{128}{3}-40}^{8/3}-48\right)=59-\frac{7}{8}-\frac{8}{3}=59-\frac{21+64}{24}=55\frac{11}{24} \end{aligned}$$

Good luck!