Midterm I PRACTICE Exam Solution

MA241-011
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Name: ____________________________

Read all of the following information before starting the exam:

- Complete each problem. Show all of your work clearly and in order and justify your answers, as partial credit will be given when appropriate and there may be NO credit given for problems without supporting work.
- **Circle or otherwise indicate your final answers..** All answers should be completely simplified, unless otherwise stated.
- **You may use a *non graphical* calculator.**
- This test has 3 problems - you have to solve them all.
- It is your responsibility to make sure that you have all of the pages!
- Good luck!

Problem	Score
1	
2	
3	
TOTAL	

1. (a) Let $f(x) = \sqrt{2x}$ and $g(x) = x^2 - 1$. Let A be domain of definition of $f(g(x))$. Let

$$h(x) = \begin{cases} f(g(x)) & x \in A \\ g(f(x)) & \text{else} \end{cases}$$

Determine whether $h(x)$ is continuous in $\mathbb{R}$ or not .

Solution:
$f(g(x)) = f(x^2 - 1) = \sqrt{2(x^2 - 1)}$, the domain is $A = \{|x| \geq 1\}$.
$g(f(x)) = g(\sqrt{2x}) = 2x - 1$
Therefore

$$h(x) = \begin{cases} \sqrt{2(x^2-1)} & |x| \geq 1 \\ 2x - 1 & \text{else} \end{cases} = \begin{cases} 2x - 1 & -1 \leq x \leq 1 \\ \sqrt{2(x^2-1)} & \text{else} \end{cases}$$

The function $h(x)$ has discontinuity at $x = 1$, since

$$\lim_{x \to 1^+} h(x) = \sqrt{2(1^2 - 1)} = 0 \neq 1 = 2 \cdot 1 - 1 \lim_{x \to 1^-} h(x)$$

Furthermore, the function $h(x)$ has also discontinuity at $x = -1$, since

$$\lim_{x \to -1^-} h(x) = \sqrt{2((-1)^2 - 1)} = 0 \neq -3 = 2 \cdot (-1) - 1 \lim_{x \to -1^+} h(x)$$

Therefore $h(x)$ is a discontinuous function in $\mathbb{R}$.

(b) Let $f(x) = \int_0^{g(x)} \frac{1}{\sqrt{1+t^3}} dt$ where $g(x) = \int_0^{\cos x} 1 + \sin t^2 dt$. Find $f'(\pi/2)$.

Solution:

$$f'(x) = \frac{g'(x)}{\sqrt{1 + g^3(x)}} = \frac{-(1 + \sin\cos^2 x)\sin x}{\sqrt{1 + g^3(x)}} = \Big|_{\pi/2} = -1$$

since $g(\pi/2) = 0$

2. (a) $\left(x^4 f\left(x\right)\right)^{(3)} =$

Solution: $= \left[12x^2 f + 8x^3 f' + x^4 f"\right]' = 24xf + \left(12+24\right)x^2 f' + \left(8+4\right)x^3 f" + x^4 f^{(3)}$.

(b) $\left[\left(\sqrt{x^2+1} + \frac{x}{\sqrt[3]{x}+1}\right)^{7/3}\right]' =$

Solution:
$= \frac{7}{3}\left(\sqrt{x^2+1} + \frac{x}{\sqrt[3]{x}+1}\right)^{4/3}\left(\frac{x}{\sqrt{x^2+1}} + \frac{x^{1/3}+1-\frac{1}{3}x^{1/3}}{\left(x^{1/3}+1\right)^2}\right)$

(c) $\int \frac{3}{\sqrt[3]{x^5}} + \frac{1}{x^3} dx =$

Solution:

$$= \int 3x^{-\frac{5}{3}} + x^{-3} dx = -9x^{-\frac{1}{3}} - \frac{1}{4}x^{-4}$$

(d) $\int \frac{x}{\sqrt{3x+5}} dx =$

Solution:

$$=_{t=x+5/3}= \frac{1}{\sqrt{3}} \int \frac{t-\frac{5}{3}}{\sqrt{t}} dt = \frac{1}{\sqrt{3}} \int \sqrt{t} - \frac{5}{3}t^{-1/2} = \frac{1}{\sqrt{3}}\left(\frac{2}{3}t^{\frac{3}{2}} - \frac{5}{6}\sqrt{t}\right)$$

3. (a) Find the length of the curve $y = \int_1^x \sqrt{\sqrt{t} - 1}dt$, where $1 \leq x \leq 16$.

Solution: We need to find $\frac{dy}{dx}$ so using the defined y and the Fundamental Theorem of Calculus, we have

$$\frac{dy}{dx} = \frac{d}{dx}\int_1^x \sqrt{\sqrt{t} - 1}dt = \sqrt{\sqrt{x} - 1}.$$

So then we use the following formula for arc length to get

$$L = \int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2}dx = \int_1^{16} \sqrt{1 + (\sqrt{x} - 1)}dx = \int_1^{16} x^{1/4}dx = \left[\frac{4}{5}x^{5/4}\right]_1^{16} = \boxed{\frac{124}{5}}$$

(b) Compute the average value of the function $f(x) = \sin \pi x$ over the interval $[0, 0.5]$.

Solution:

$$f_{avg} = \frac{1}{b-a}\int_a^b f(x)dx = \frac{1}{0.5 - 0}\int_0^{0.5} \sin \pi x dx = -\frac{2}{\pi}\cos \pi x\Big|_0^{0.5} = \frac{2}{\pi}$$

(c) Find force required to compress a spring from its natural lenght of 1ft to a length of 0.75ft if the spring constant $k = 16$.

Solution: The force is $f(x) = 16x$, the displacement is 0.25 so the work is

$$W = \int_0^{0.25} 16x dx = 0.5J$$