

1. Determine whether the following integrals converge or diverge.

(a)  $\int_1^{\infty} \frac{\sin(x) \cos(x)}{x^{3/2}} dx$

(b)  $\int_5^{\infty} \frac{x}{x^2 + 5} dx$

2. Let  $f(x) = \frac{1}{x}$ .

- (a) Calculate the volume of the solid of revolution of  $f$ , rotating  $f$  about the  $x$ -axis, on the interval  $x \in [1, \infty)$ . (recall that the formula is  $V = \pi \int_a^b f(x)^2 dx$ )
- (b) The surface area of a solid of revolution constructed by rotating  $y = f(x)$  about the  $x$  axis is given by  $SA = 2\pi \int_a^b f(x) \sqrt{1 + f'(x)^2} dx$  (note the similarity to the arc length formula!). Calculate the surface area of the solid of revolution of  $f$ , rotating  $f$  about the  $x$ -axis, on the interval  $x \in [1, \infty)$ .

3. Determine the following improper integrals

(a)  $\int_{-\infty}^{\infty} xe^{-x^2} dx$

(b)  $\int_{-\infty}^{\infty} \sin(x) dx$

(c)  $\int_0^5 \frac{1}{\sqrt{5-x}} dx$

(d)  $\int_0^{\infty} \frac{1}{x} dx$