Extra Assignment

1 vectors and vector functions

1. Given $\vec{v}_1 = \langle 1,3,4\rangle$ and $\vec{v}_2 = \langle \pi, e, 7\rangle$, find
   (a) the distance from v_1 to v_2,
   (b) $v_1 \cdot v_2$ and $v_1 \times v_2$,
   (c) the (parametric) equation for a line through the points $(1,3,4)$ and $(\pi, e, 7)$,
   (d) the equation for the plane containing the points $(1,3,4)$, $(\pi, e, 7)$ and the origin.
2. Calculate the circumference of a circle by parametrizing the circle and using the arc length formula.

2 derivatives and optimization

1. Find an equation for the tangent plane to the ellipsoid $x^2/a^2 + y^2/b^2 + z^2/c^2 = 1$ at the point $p = (a/\sqrt{3}, b/\sqrt{3}, c/\sqrt{3})$.
2. Find the local maximums and minimums of $f(x,y) = \sin(x)\sin(y)$ for $0 \le x \le \pi$ and $0 \le y \le \pi$.
3. Let $f(x,y) = 4 - x^2 - y^2$. Find points on the ellipse E defined by $x^2/4 + y^2 = 1$ where f is a maximum and where f is a minimum.
4. Let $f(x,y) = 4 - x^2 - y^2$. Find points in the region D defined by $x^2/4 + y^2 \le 1$ where f is a maximum and where f is a minimum.

3 double and triple integrals

1. Use triple integral to find volume of given solid.
   (a) The solid enclosed by the cylinder $x^2+y^2 = 9$ and the planes y+z=5 and z=1
   (b) The solid enclosed by the paraboloid x $=z^2+y^2$ and the plane x=16
2. Write integrals which give the following (no need to evaluate):
   (a) the center of mass of the region contained in the ellipsoid $x^2/a^2+y^2/b^2+z^2/c^2 = 1$ which is below the z-axis, assume density is constant.
   (b) the volume of the region which is contained in both the cylinder of radius 2 centered on the z-axis and the sphere of radius 5 centered at the origin.
   (c) the average value of the function $f(x,y,z) = x^2y + z$ over the region W, where W is the part of the unit sphere with $x \ge 0$ and $y \ge 0$.

4 vector calculus

1. Calculate the following line integrals $\int_C \phi(\vec{r}) \cdot d\vec{r}$:

   (a) $\phi(x,y) = xy\mathbf{i} + y^2\mathbf{j}$ and C is the piece of the parabola $y = x^2$ which goes from the origin to the point $(1,1)$.

   (b) $\phi = 2xy^2\mathbf{i} + 2x^2y\mathbf{j}$ and C is the circle of radius 100000000000000.

   (c) $\phi = -y^3\mathbf{i} + x^3\mathbf{j}$ and C is the unit circle (use a theorem...).

2. For each of the following vector fields Φ, decide whether or not they are conservative. If they are, find a function F so that $\nabla F = \Phi$:

   (a) $\Phi(x,y) = xy^2\mathbf{i} + xy\mathbf{j}$.

   (b) $\Phi(x,y,z) = 2xy^2z^2\mathbf{i} + 2x^2yz^2\mathbf{j} + 2x^2y^2z\mathbf{k}$.

   (c) $\Phi(x,y,z) = 3x^2y^2z\mathbf{i} + 2x^3yz\mathbf{j} + 3x^2y^2\mathbf{k}$.

3. Calculate the divergence and curl of the previous vector fields. Recall the relationship between curl and whether or not a vector field is conservative.

4. Calculate the flux $\int_C \Phi \cdot \vec{n} ds$ of the following vector fields:

   (a) $\Phi = 2y^2\mathbf{i} + x^2\mathbf{j}$ and C is the part of the unit circle going from $(1,0)$ to $(0,1)$.

   (b) $\Phi = x^3y^2\mathbf{i} - \sin(y)\mathbf{j}$, and C is the rectangle formed by lines at $x = -2$, $x = 1$, $y = 1$, $y = 3$ (use a theorem...).

   (c) $\Phi = x^2\mathbf{i} + y^2\mathbf{j}$ and C is the unit circle.

5. Evaluate the surface integral $\iint_S y dS$ where S is helicoid with vector equation $\mathbf{r}(u,v)=<u \cos v, u \sin v, v>$, $0 \le u \le 1$, $0 \le v \le \pi$

6. Verify that Stoke's Theorem is true for given verctor field **F** and surface S

   (a) $\mathbf{F}=<-y,x,-2>$, and S is the cone $z^2=x^2+y^2$, $0 \le z \le 4$ oriented downward

   (b) $\mathbf{F}=<y,z,x>$, and S is the hemisphere $x^2+y^2+z^2=0$, $y \ge 0$ oriented in the direction of the positive y-axis.

7. Verify that the Divergence theorem is true for the vector field **F** on the region E

   (a) $\mathbf{F}=<3x,xy,2xz>$, and E is the cube bounded by the planes x=0, x=1, y=0, y=1, z=0 and z=1.

   (b) $\mathbf{F}=<x^2,-y,z>$, and E is the solid cylinder $x^2+z^2 \le 9$, $0 \le x \le 2$.

8. The Fundamental Theorem of Calculus always says roughly: Given a region R whose boundary is B, the integral (i.e. a normal integral, line integral, surface integral, multiple integral) of "something" (i.e. a function, 2D vector field, 3D vector field) over B is equal to the integral of "the derivative of that something" (i.e. the regular derivative, the gradient, the curl, or the divergence) over R. The different theorems we saw in last part of the course are all of this form, its just that the something, the integral, and the derivative all take various different forms. Write out each of the fundamental theorems seen in chapters 6 and 7, as well as the standard fundamental theorem from last semester, and in each, say what kind of region R you have, what its boundary B looks like, what types of integrals you're calculating, and what the derivative means.