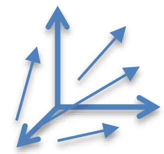
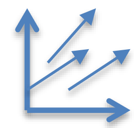
Handout 2

Microsoft Mathematics: a software for 3D visualization – useful tool
http://www.microsoft.com/en-us/download/details.aspx?id=15702

Definition: Vector is mathematical object that specify length/magnitude and direction. We denote a vector by putting a little over on the top, or sometime in books by using bold font. $\vec{a},\vec{u},\vec{v}$ or $\mathbf{a},\mathbf{u},\mathbf{v}$. We write a vector as a list of numbers in angle brackets $\langle x,y\rangle$ or $\langle x,y,z\rangle$.





Denote $\overrightarrow{AB}$ a vector from point A to point B. One sketches $\overrightarrow{AB}$ as a line segment with an arrow that point in the direction of the vector. However, the vector considered as a numerical quantity that doesn't represent the locations of A and B, but the location of point B relative to A as if A were the coordinate origin. Thus, vectors are position-less and therefore:

Definition: two vectors considered equivalent if their magnitude\length and direction is equal, regardless the position. Similarly $\langle x_1,y_1,z_1\rangle=\langle x_2,y_2,z_2\rangle$ $\langle x_1,y_1\rangle=\langle x_2,y_2\rangle$ iff $x_1=x_2, y_1=y_2, z_1=z_2$.

Definition: A vector from point $A(x_1,y_1,z_1)$ ($\tilde{A}(x_1,y_2)$) to point $B(x_2,y_2,z_2)$ ($\tilde{B}(x_2,y_2)$) is denoted as $\overrightarrow{AB}=\langle x_2-x_1, y_2-y_1, z_2-z_1\rangle$ ($\overrightarrow{\tilde{A}\tilde{B}}=\langle x_2-x_1, y_2-y_1\rangle$)

Vector Arithmetic

Let $\vec{a}=\langle a_1,a_2\rangle,\vec{b}=\langle b_1,b_2\rangle,\vec{c}=\langle c_1,c_2,c_3\rangle,\vec{d}=\langle d_1,d_2,d_3\rangle$ be a vectors and *k* a constant. In order to distinct vectors and numbers we will call *k* a **scalar**.

Definition: Defined the length or a magnitude of 2D, 3D vectors $\vec{a},\vec{c}$ respectively by $\|\vec{a}\|=\sqrt{a_1^2+a_2^2}$ and $\vec{c}=\sqrt{c_1^2+c_2^2+c_3^2}$ respectively.

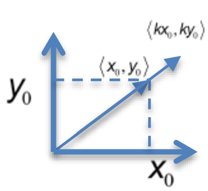
Definition: The **unit vector** is a vector with magnitude\length equal 1.

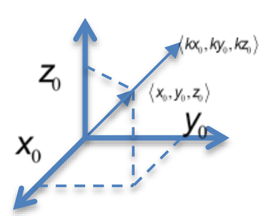
Definition: Scalar Multiplication defined as $ka=\langle ka_1,ka_2\rangle$ and $kc=\langle kc_1,kc_2,kc_3\rangle$. If k<0 then the vector change direction. Note also that

$$\|k\vec{a}\|=\sqrt{k^2a_1^2+k^2a_2^2}=|k|\sqrt{a_1^2+a_2^2}=|k|\|\vec{a}\| \text{ and}$$

$$\|k\vec{c}\|=\sqrt{k^2c_1^2+k^2c_2^2+k^2c_3^2}=|k|\sqrt{c_1^2+c_2^2+c_3^2}=|k|\|\vec{c}\|$$

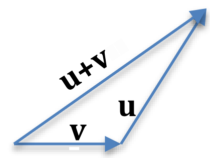
Thus scalar multiplication scales (either stretch or squeeze) vector magnitude\length.

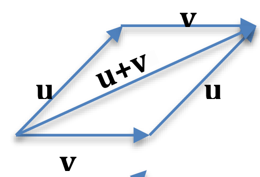




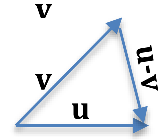
Note: Every vector **u** can be converted into a unit vector by multiplying it by very special scalar equal to reciprocal of its magnitude: $\left\| \frac{1}{\|\vec{u}\|}\vec{u} \right\| = \left| \frac{1}{\|\vec{u}\|} \right| \|\vec{u}\| = \frac{\|\vec{u}\|}{\|\vec{u}\|} = 1$.

Definitino: **Vector Addition** defined as $\vec{a}+\vec{b}=\langle a_1+b_1, a_2+b_2\rangle$ and $\vec{c}+\vec{d}=\langle c_1+d_1, c_2+d_2, c_3+d_3\rangle$. The addition of vectors can be understood geometrically as as a Triangle Law or a Parallelogram Law, which are illustrated below.





Definition: **Vector Subtraction** defined by $\vec{u}-\vec{v}=\vec{u}+(-\vec{v})$ where $-\vec{v}$ is the vector with the same magnitude but opposite direction $\vec{v}$.



Properties of vectors:

1. $\vec{a}+\vec{b}=\vec{b}+\vec{a}$
2. $(\vec{a}+\vec{b})+\vec{c}=\vec{a}+(\vec{b}+\vec{c})$
3. $\vec{a}+\vec{0}=\vec{a}$
4. $\vec{a}+(-\vec{a})=\vec{0}$
5. $k(\vec{a}+\vec{b})=k\vec{a}+k\vec{b}$
6. $(k+m)\vec{a}=k\vec{a}+m\vec{a}$
7. $(km)\vec{a}=k(m\vec{a})$

Standard basis (i-j-k notations)

We define 3 very important vectors, called a **Standard Basis:**
$\mathbf{i}=\langle 1,0,0\rangle$; $\mathbf{j}=\langle 0,1,0\rangle$; $\mathbf{k}\langle 0,0,1\rangle$ in 3D or equivalently $\mathbf{i}=\langle 1,0\rangle$; $\mathbf{j}=\langle 0,1\rangle$ in 2D.

Theorem: Every vector can be expressed as a sum of vectors of the standard basis as following

$$\langle a_1,a_2,a_3\rangle = a_1\langle 1,0,0\rangle + a_2\langle 0,1,0\rangle + a_3\langle 0,0,1\rangle = a_1\mathbf{i}+a_2\mathbf{j}+a_3\mathbf{k}$$
$$\langle a_1,a_2\rangle = a_1\langle 1,0\rangle + a_2\langle 0,1\rangle = a_1\mathbf{i}+a_2\mathbf{j}$$